Solvency II: It's not Normal

The Normal, or Gaussian, probability distribution is mathematically the easiest to aggregate into a portfolio or business risk distribution. Therefore the temptation is always there for a modeler to assume that risks factors follow a Normal distribution – it makes the modeling easier. The trouble is that Nassim Taleb's black swan events do not really exist in a Normal distribution. BRAVE Partners reviews a mathematical framework published ten years ago in Risk Magazine that is both straightforward to implement, but allows an abundance of black swan events.

The issue of portfolio risk and optimisation with non-Normal distributions has popped up on LinkedIn frequently over the past few weeks. BRAVE Partner, Christopher Cloke-Browne, worked with a team that developed a solution ten years ago. The insight from that work is still relevant today.

Abnormal

The problem with using a Normal distribution for risk management is almost spelt out in its name. Risk management is about managing for the abnormal events – the stress events. The Normal distribution is a great representation of the normal, or every day, events and gives up a representation of stress events for simplicity.

Stress and hassle

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However, to add stress scenarios to a risk model, the modeller has to give up the assumption that the underlying risks, or variables, are normally distributed. This then poses a challenge.

Normal is simple

Risk modelling is about aggregating risks and dependencies to get an overall value distribution for a portfolio or business. The attraction of a Normal distribution is the fact that it is mathematically straightforward to aggregate them together. This simplicity persists even when the distributions have some form of dependence – or in the case of Normal distributions – correlation. Non-Gaussian distributions are far more complex to aggregate. Adding a correlation of dependency structure on top of that is harder still.

Condos on Mars

BRAVE Partners posted a piece on risk modelling entitled "Don't build a condo on mars" in honour of the witty comment of an ABS trader expressing his views of correlation trading – or the synthetic CDO business that was highly popular in European banks. The ABS trader's view was that buying correlation was similar to buying a condo on Mars. He was sure that he would be able to buy a condo on Mars one day, but quite what it would look like or what it would be worth was still pretty unclear to him.

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The quote is both amusing and highly accurate. Mathematically, correlation is just not properly defined for the binary distributions that represent defaults in underlying CDO assets. This can be clearly seen by just considering what the correlation is between two default loss distributions; one with a default probability of 5% and one with a default probability of 10%. If it is observed that both assets always default together then it would seem natural to say that the correlation is 100%. However the calculation of the correlation will not produce this answer. Drawing out the default events will reveal why. Since the 10% default rate asset must default more often than the 5% asset there must be occasions where the 10% default rate asset defaults and the 5% does not – so the correlation cannot be 100%. The situation gets more complex if a third asset with a 2% default rate is added. A whole portfolio of assets with different default rates is enough to make anyone's head spin!

Domain change

The trick to aggregating dependent non-Normal distributions lies in a domain change. The usual representation of a random variable, for these purposes the asset value, is a probability density function, or PDF. For a Normal distribution this is the classic bell shaped curve. However, all distributions

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Magic moments

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Splat!

The world of MGFs is close to mathematical paradise. However, much like paradise, in the end reality bites. The problem with the MGF domain is that it is all very easy to combine and manipulate distributions. However, it is hard to deduce the actual probability of a particular loss from the MGF – for that you need the PDF. Transforming from an MGF to a PDF – now that is mathematical hard core!

Sneakers

Fortunately there is a mathematical technique that provides an accurate approximation of the reverse transformation for MGF to PDF called the saddle point technique. The application of the saddle point technique to this issue was published in Risk Magazine by Martin, Thompson and Browne. The articles was entitled “Taking to the Saddle”.

Applications

Mathematical niceties such as the ones described are seen as beautiful by mathematicians and indeed there is some satisfaction in developing a neat solution to an issue. However, in a commercial world such solutions are only as good as the business applications that they can generate.

Portfolio risk and optimisation

BRAVE Partner, Christopher Cloke-Browne developed the risk modeling framework described here into a credit portfolio advisory business at one of the major investment banks. The power of the framework was that it would allow automated portfolio optimisation.

One of the greatest failings of a Normal distribution is that it does not penalize risks that can cause outlier events – black swans. Markets tend to have some understanding of this issue and so the risk will trade at a premium in the market. Portfolio optimization that takes a market price for the risk, but does not (or more accurately – *can* not) fully represent the risk will always look to load up the portfolio in the risky asset. The excess spread for the excess risk looks like free money in this framework.

Sub prime and ABS

No discussion of risk would be complete without a mention of sub prime. The issue identified above is a part of the circumstances that lead to the collapse of that market and subsequent credit crisis. Investors bought the senior tranches of sub prime ABS because they were rated AAA, but paid a higher spread than AAA bonds – eg GE was AAA at that time. Investors bought these bonds because the risk was hard to model – so they relied on the rating. The rating was set by looking at the risk to the bond under normal conditions. The ratings did not account for the cliff risk of large downside moves in the market. In short, the expected outcome from a GE bond and an AAA sub-prime ABS bond are the same. But in extreme conditions – a black swan event – the outcomes are very different. Investors failed to appreciate this difference and thus saw the increased yield on the asset as free money. Risk modeling in the BRAVE Partners framework makes this type of risk explicit.

BRAVE Partners services

BRAVE Partners can assist clients in developing a simple, sophisticated risk modeling framework. This framework will capture the key risk elements for an asset portfolio or a business.

Risk budgeting and decision making will be brought together in a way that is straightforward and links the discussions of management to the business.

Solvency II demands that model and governance are integrated. If your existing model does not look like your business then this will not be easy for you. The BRAVE Partners approach to risk modeling enables the production of a transparent framework that meets all of the demands of Solvency II.

- *Value distributions for risk have to account for outlier events – Nassim Taleb famously calls these black swan events.*
- *Aggregating dependent non-Normal distributions is mathematically complex. A risk model to account for black swans is therefore complex.*
- *BRAVE Partners is well versed in a straightforward transparent method for analysing black swan event risks.*

- *BRAVE Partners can assist insurers and asset managers to build a powerful modeling framework.*
- *This framework is straightforward to integrate with the language and decision making process of the business management.*
- *The framework is thus ideal for Solvency II.*

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Interaction

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